PID Controller Design for a Real Time Ball and Beam System – A Double Integrating Process with Dead Time

I.Thirunavukkarasu[1], Marek Zyla[2], V.I.George[3]and Shanmuga Priya[4]

[1 & 4]Associate Professor, Dept. of ICE & Chemical Engg, MIT

[3]IAESTE Student, AGH Univ. of Science and Tech, Poland.

[2]Registrar, Manipal University Jaipur, Rajasthan.

Mail: it.arasu@manipal.edu

***Abstract*— In this paper, the authors have discussed and shown how to tune the PID controller in closed loop with time-delay for the double integrator systems for a particular stability margins. In math model it is assumed that time delay (ô) of the plant is known. As a case study the authors have considered the mathematical model of the real-time beam and ball system and analyzed the simulation and real time response.**

***Index Terms*— double integrator, PID, stability, time delay**

I. Introduction

JWatkins [1] worked with the PD control for double integrator systems with time delay. This paper is an extension in which PID control is analyzed in simulation as well as in real time. Integral part of the controller eliminates steady-state error, which can be necessary in this kind of systems.

Equations delivered in this paper and m-files based on them can be helpful in tuning a PID controlled real-time model of beam and ball system, which is an example of double integrator system with time delay.

II. Stability

Consider the feedback control system shown in Fig. 1. The closed-loop transfer function can be written as

$$T(s) = \frac{sK_p + K_i}{\frac{s^3}{m} + sK_p e^{-s\tau} + K_i e^{-s\tau} + s^2 K_d e^{-s\tau}} \tag{1}$$

or

$$T(\tilde{s}) = \frac{\tilde{K}_i + \tilde{s}\tilde{K}_p}{\tilde{s}^3 + \tilde{s}\tilde{K}_p e^{-\tilde{s}} + \tilde{K}_i e^{-\tilde{s}} + \tilde{s}^2 \tilde{K}_d e^{-\tilde{s}}} \tag{2}$$

where

$$\tilde{s} = s\tau \tag{3}$$

$$\tilde{K}_i = K_i m \tau^3 \tag{4}$$

$$\tilde{K}_p = K_p m \tau^2 \tag{5}$$

$$\tilde{K}_d = K_d m \tau \tag{6}$$

The characteristic equation of system (2) can be written as

$$1 + L(\tilde{s}) = 1 + \frac{\tilde{s}\tilde{K}_p + \tilde{s}^2\tilde{K}_d + \tilde{K}_i}{\tilde{s}^3} e^{-\tilde{s}} \tag{7}$$

By setting $\tilde{\omega} = \omega\tau$ magnitude and phase of $L(j\tilde{\omega})$ in frequency domain can be written as

$$\left|L(j\tilde{\omega})\right| = \sqrt{\frac{\tilde{K}_p^2}{\tilde{\omega}^4} + \left(\frac{\tilde{K}_d}{\tilde{\omega}} - \frac{\tilde{K}_i}{\tilde{\omega}^3}\right)^2} \tag{8}$$

$$\angle L(j\tilde{\omega}) = \tan^{-1}\left[\frac{\tilde{\omega}^2\tilde{K}_d - \tilde{K}_i}{\tilde{\omega}\tilde{K}_p}\right] - \tilde{\omega} - \pi \tag{9}$$

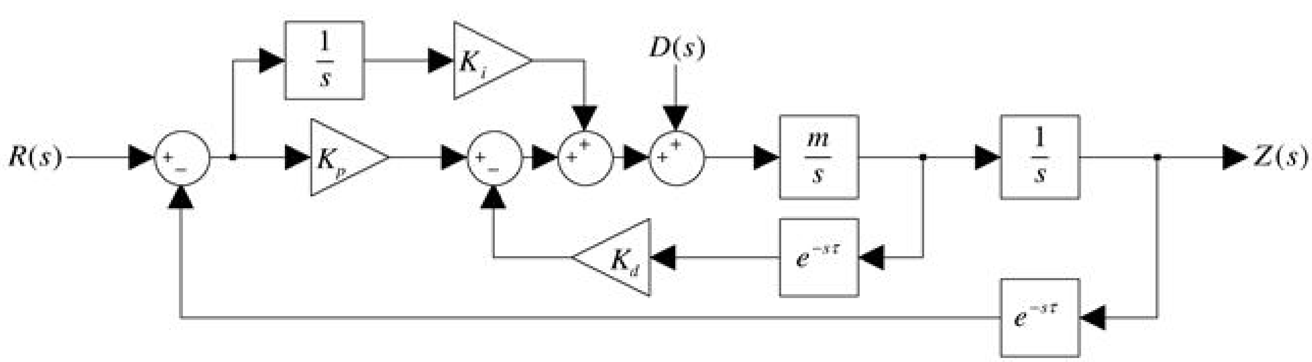

Fig. 1. Feedback control system with PID controller and double integrating plant with time delay. It is assumed that velocity (derivative of controlled value) is known.

DOI: 03.LSCS.2013.3.511

At gain crossover frequency

$$\left| L(j\tilde{\omega}_{gc}) \right| = 1 \tag{10}$$

$$PM = \angle L(j\tilde{\omega}_{gc}) + \pi \tag{11}$$

Substituting (9) into (11) gives

$$PM = \tan^{-1}\left[\frac{\tilde{\omega}^2 \tilde{K}_d - \tilde{K}_i}{\tilde{\omega}\tilde{K}_p}\right] - \tilde{\omega} \tag{12}$$

Rearranging (12) it can be shown that

$$0 < PM + \tilde{\omega}_{gc} < \frac{\pi}{2} \tag{13}$$

Using (8), (10), and (12) it can be shown that

$$\tilde{K}_p = \frac{\tilde{\omega}_{gc}}{\sqrt{1+\tan^2(PM + \tilde{\omega}_{gc})}} \tag{14}$$

$$\tilde{K}_i(\tilde{K}_d) = \tilde{\omega}_{gc}^2\left(\tilde{K}_d + \tilde{\omega}_{gc}\sqrt{1-\cos^2(PM + \tilde{\omega}_{gc})}\right) \tag{15}$$

By setting $\tilde{K}_d$ as a parameter, (14) and (15) can be used to choose phase margin of the system. The stability boundary (as far as phase is concerned) can be plotted by setting $PM = 0$.

At the phase crossover frequency

$$GM = \frac{1}{\left| L(j\tilde{\omega}_{pc}) \right|} \tag{16}$$

$$\angle L(j\tilde{\omega}_{pc}) = \pi \tag{17}$$

Substituting (9) into (17) gives

$$\tilde{\omega}_{pc} = \tan^{-1}\left[\frac{\tilde{\omega}_{pc}^2 \tilde{K}_d - \tilde{K}_i}{\tilde{\omega}_{pc}\tilde{K}_p}\right] \tag{18}$$

From (18) it can be shown that

$$0 < \tilde{\omega}_{pc} < \frac{\pi}{2} \tag{19}$$

Using (8), (16), and (18) it can be shown that

$$\tilde{K}_p = \frac{\tilde{\omega}_{pc}^2}{GM\sqrt{1+\tan^2\tilde{\omega}_{pc}}} \tag{20}$$

$$\tilde{K}_i(\tilde{K}_d) = \left(\frac{\tilde{K}_d}{\tilde{\omega}_{pc}} - \sqrt{\frac{1}{GM^2} - \frac{\tilde{K}_p^2}{\tilde{\omega}_{pc}^4}}\right)\tilde{\omega}_{pc}^3 \tag{21}$$

By setting $\tilde{K}_d$ as a parameter, (20) and (21) can be used to choose gain margin (GM) of the system. The stability boundary (for gain) can be plotted by setting $GM = 1$.
Substituting (3), (4), (5), and (6) into (13), (14), (15), (19), (20), and (21) gives

$$0 < PM + \tau\omega_{gc} < \frac{\pi}{2} \tag{22}$$

$$K_p = \frac{\omega_{gc}}{m\tau\sqrt{1+\tan^2(PM + \tau\omega_{gc})}} \tag{23}$$

$$K_i(K_d) = \frac{\omega_{gc}^2\left(K_d m + \omega_{gc}\sqrt{1-\cos^2(PM + \tilde{\omega}_{gc})}\right)}{m} \tag{24}$$

for phase margin plot and

$$0 < \omega_{pc} < \frac{\pi}{2\tau} \tag{25}$$

$$K_p = \frac{\omega_{pc}^2}{mGM\sqrt{1+\tan^2\tau\omega_{pc}}} \tag{26}$$

$$K_i(K_d) = \left(\frac{\tilde{K}_d}{\tilde{\omega}_{pc}} - \sqrt{\frac{1}{GM^2} - \frac{m^2K_p^2}{\omega_{pc}^4}}\right)\frac{\omega_{pc}^3}{m} \tag{27}$$

for gain margin plot.

Combining PM and GM plots for ω that satisfy (22) and (25) stability margins for both phase and gain can be observed for fixed τ, m, and K_d as on Fig. 2 and 3. Fig.

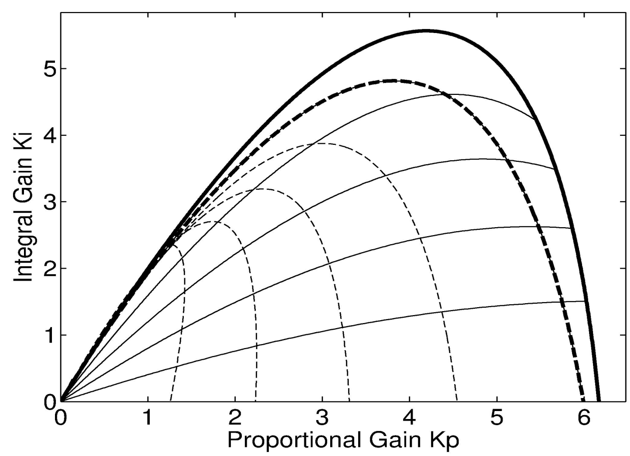

Fig. 2. Stability margins of the system. Solid line states for gain and dashed line for phase. Bold lines are the borders of stability. Below both bold lines the system is stable. Each step from higher dashed line to a lower one increases phase margin by $\pi/20$. Each step from higher solid line to a lower one increases gain margin by 0.2. The highest stability margins are obtained in lower left corner.

DOI: 03.LSCS.2013.3.511

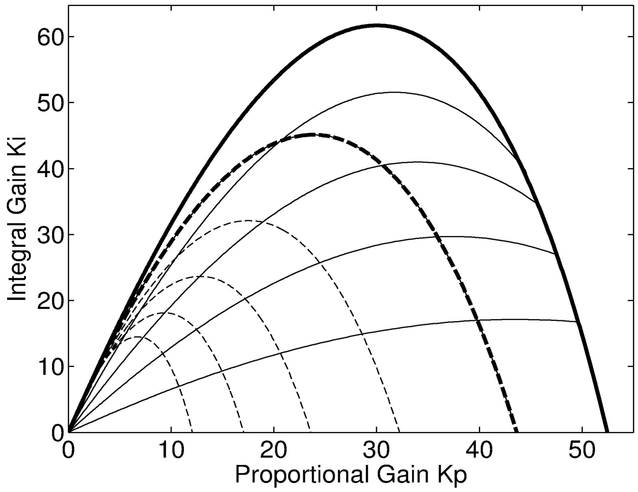

Fig. 3. Stability margins of the system. Solid line states for gain and dashed line for phase. Bold lines are the borders of stability. Below both bold lines the system is stable. Stability margins are presented as on Fig. 2

2 presents plot for $\tau = 0.35$, $K_d = 3$ and $m = 0.7$. Fig. 3 depicts plot for $\tau = 0.1$, $K_d = 5$ and .

III. Simulation

Simulation of the control process was made using Matlab Simulink software [2-4]. A model was created as on Fig. 1 and a simulation was run with 2 sets of parameters. Fig. 4 and 5 show step and load disturbance responses of the closed-loop system for this two sets of parameters. Load disturbance was a 0.2 step.

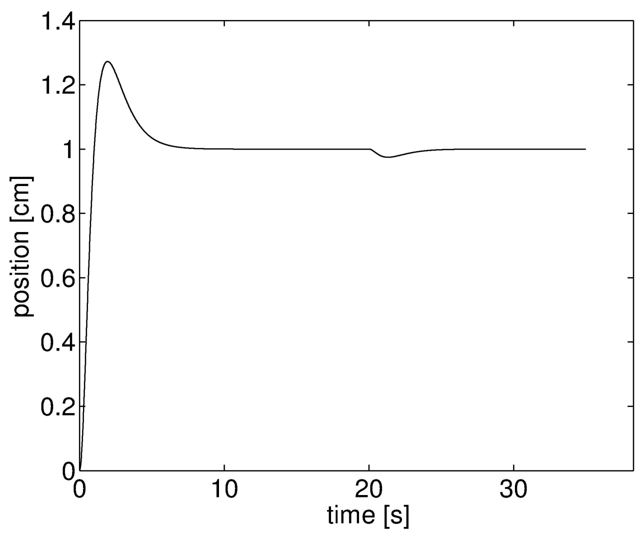

Fig. 4. Step and load disturbance response of the closed-loop system for parameters: $K_p = 6.8$, $K_i = 3.1$, $K_d = 5$, $\tau = 0.1$.

IV. Real-Time Example

The machine used for real-time experiment was Googoltech Ball & Beam digital control system. Transfer function of the model of the system can be written as

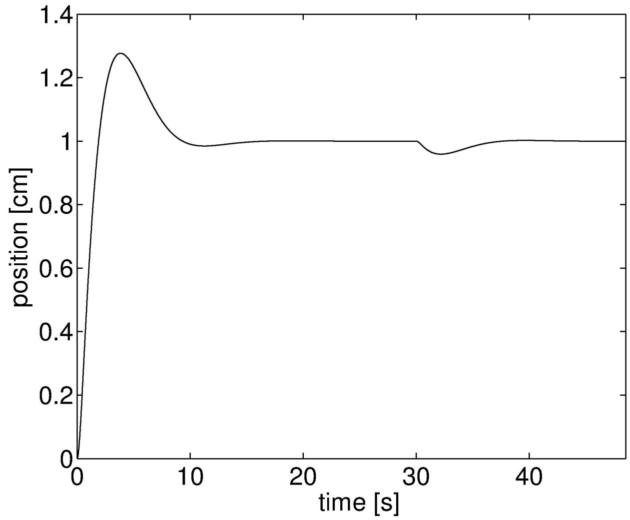

Fig. 5. Step and load disturbance response of the closed-loop system for parameters: $K_p = 3.5$, $K_i = 1.3$, $K_d = 5$, $\tau = 0.1$.

$$T(s) = \frac{0.7}{s^2} e^{-s\tau} \tag{28}$$

Control was realized by Matlab Simulink, by an already prepared manufacturer model. The user's task was only to put PID gain parameters.

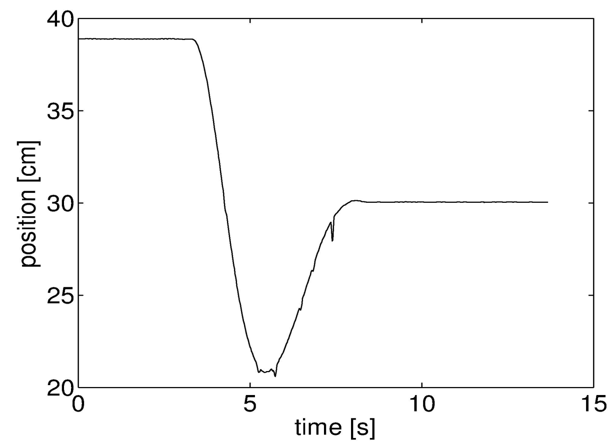

Fig. 6. Step response of the real-time system

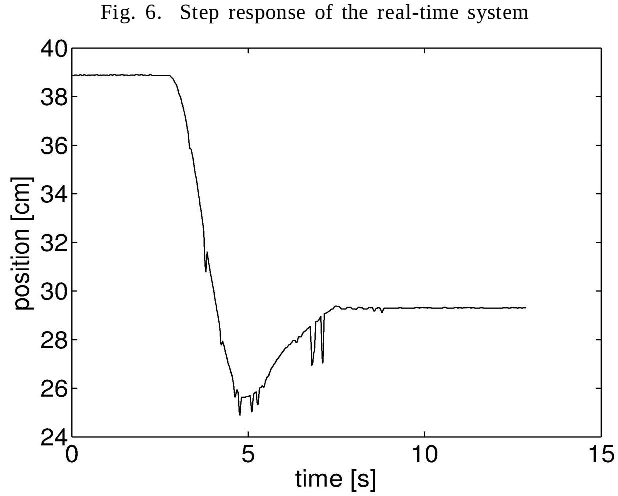

Fig. 7. Step response of the real-time system

DOI: 03.LSCS.2013.3.511

Step responses are shown on Fig. 6 and 7, and load disturbance response is presented on Fig. 8.

Experiment was made with parameters:

$$K_p = 6.8 \tag{29}$$

$$K_i = 3.1 \tag{30}$$

$$K_d = 5 \tag{31}$$

It was observed that results of the experiment differ slightly from ones obtained in the simulation. Overshoot in the real-time was clearly higher and results were not repeatable. The main reason is low quality of the equipment. The ball would become stuck on the beam what made the model inadequate to reality. Furthermore, the time delay in simulation was chosen experimentally and it may be inaccurate.

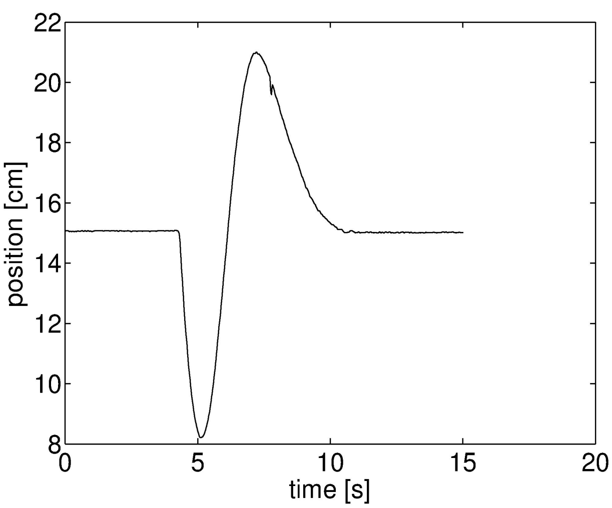

Fig. 8. Load disturbance response of the real-time system

Acknowledgment

The authors thank MIT, Manipal University for providing the facility for the real time experimentation and the authors express the gratitude to Dr.Shreesha, Head of the Dept. ICE and to Mr.Santosh Kumar Chowdhary, In-Charge Space Engineering Lab, ICE Dept, MIT, Manipal.

References

[1] J. Watkins, G. Piper, J. Leitner "Control of Time-Delayed Double Integrator Systems", Proceedings of the American Control Conference, Denver, Colorado, 2003, pp. 1506–1511.

[2] Tuyres & Luyben Tuning PI controller for Integral / Time delayed process, *Industrial Engineering chemical research. Vol.31. P. No: 2625-2628,* 1992.

[3] R.Padma Shree and M.Chidambaram "Control of Unstable System", *Narosa Publications. ISBN: 978-81-7319-700.*,2005.

[4] A.Visioli, "Optimal tuning of PID controllers for integral and unstable processes", *IEE Proc.-Control Theory Appl., Vol. 148, No. 2. P.No: 180-184, 2001*

[5] I.Thirunavukkarasu, "Optimal Robust H^{∞} Controller for an Integrating Process with Dead Time", Ph.D Thesis, MIT, Manipal University, India. 2012.

DOI: 03.LSCS.2013.3.511